

## NONLINEAR ANALYSIS OF REINFORCED CONCRETE CONTINUOUS DEEP BEAMS USING STRINGER-PANEL MODEL

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### ABSTRACT

This paper describes the numerical tests for the nonlinear analysis of eight reinforced concrete (RC) deep beams. A stringer-panel model is presented for the nonlinear analysis of the deep beams. Cracked reinforced concrete is treated as an orthotropic material. To model nonlinear material response, the constitutive relations currently utilized are those of the modified compression field theory. Stiffness matrices are defined for concrete and reinforcement, and element stiffness matrices are derived for stringer and panel elements. A solution algorithm is described. The ability of the stringer-panel model to assess ultimate load is evaluated by correlation studies with available experimental data. The computational efficiency, numerical stability, and potential application of the model are demonstrated through example analyses.

**Keywords:** reinforced concrete deep beams, stringer-panel model, constitutive relation, modified compression field

### 1. INTRODUCTION

The finite and discrete element method offers a powerful and general analytical tool for studying the behavior of reinforced concrete. Cracking, tension stiffening, nonlinear multi-axial material properties, complex steel-concrete interface behavior, and other effects previously ignored or treated in a very approximate way can be modeled rationally. Through such studies, in which the important parameters may be varied conveniently and systematically, new insights are gained that may provide a firmer basis for the codes and specifications on which ordinary design is based.

The major difficulties in the nonlinear analysis of reinforced concrete arise in selecting appropriate discrete element models, constitutive relations for elastic and inelastic under combined stress state, and failure criteria for the concrete, steel, bond, and aggregate interlock. For a beam subjected to both flexure and shear, the problem is much more

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complex. A rational analytical solution to this problem, which includes the effects of cracking, bond, dowel action and aggregate interlock, and ultimate strength, still remains one of the important problems to be solved, especially for deep beams in which the regions of high shear and high moment coincide and failure usually occurs in these regions. Because of these complexities, design procedures for beams subjected to moment plus shear have been based on extensive test data. Unfortunately, these empirical procedures cannot be extrapolated to beams subjected to combined axial force, shear and moment, or to the more complicated reinforced concrete structures. These are the types of local analysis and design problems of deep beam for which nonlinear discrete element analyses may offer a solution.

Despite the fact that they are structural elements commonly found in construction, the procedures employed for designing reinforced concrete deep beams are still not firmly established [1]. The difficulties of numerical analysis originate, as mentioned above, not only from the theoretical aspect of the problem, where the interference among cracks is at the core of the problem's solution, but also from its computational aspect. Apparently, the numerical treatment of several varying crack surfaces requires a certain degree of flexibility and sophistication in the modeling techniques. This explains why a great amount of theoretical as well as experimental work has been to assess the bearing capacity of such structures in a more rational way. Most of these studies are based on the analysis of experimental results in the light of the so called "strut-and-tie" model first introduced by Ritter [2], and more recently developed by Siao [3]. However, it should be pointed out that because the interaction between concrete and reinforcement in the connecting nodes was not analyzed in detail, such approaches do not provide rigorous lower-bound estimates for the limit loads. Perhaps least practical, separate strut-and-tie models are often required for different load combinations. Generally speaking, numerical analysis procedures may yield reliable estimates for the ultimate loads, provided an appropriate discrete element model is used for such analyses.

In this paper, a discrete element model consisting of stringers and panels applied to estimating the shear failure loads of deep beams is presented. To model nonlinear material response, the constitutive relations currently utilized are those of the modified compression field theory. Stiffness matrices are defined for concrete and reinforcement, and element stiffness matrices are derived for stringer and panel elements. A solution algorithm is described. The ability of the stringer-panel model to assess ultimate load is evaluated by correlation studies with available experimental data.

## 2. TEST MODEL

### *2.1 Stringer element and panel element*

As can be observed in every day practice, the reinforcement of deep beams consists often of a mesh reinforcement at the surfaces and reinforcing bars along the edges and around holes.

Starting from this geometry, a discrete element model in which some elements called "stringers" contain main reinforcing bundles and other called "panels" contain a distributed reinforcing mesh as shown in Figure 1 is developed at Delft university of technology [4].

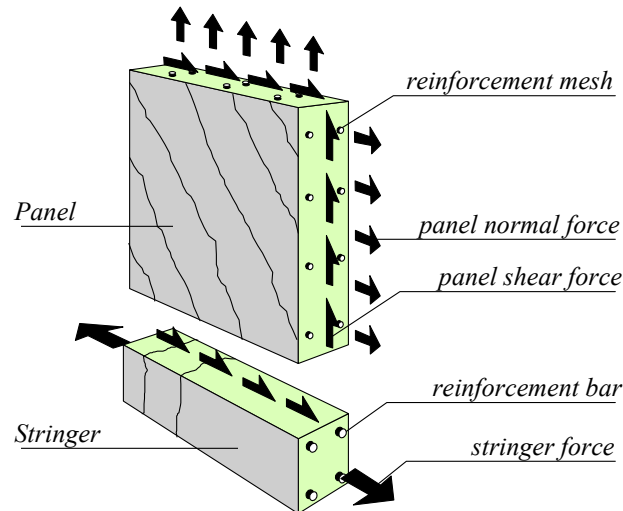


Figure 1. Stringer and panel elements

A structure is modeled as an assembly of stringers and panels which are in perfect equilibrium. The stringer elements carry only normal stresses. The panel elements used in the model have not only distributed shear tractions acting on the edges but normal tractions as well. The panel element has four edge tractions and three independent equilibrium relations. It will be shown that each traction is accompanied by a discrete edge displacement. So, the panel element has four degree of freedom too. Hoogenboom and Blaauwendraad [5] have developed a linear-elastic shear panel with a quadrilateral shape for computing stringer forces and panel stresses. The linear-elastic stringer-panel model provides a simple yet sufficiently accurate way to quantify the forces for dimensioning of a bracket components compared with a discrete element model. But, there are two practical problems with reinforced concrete deep beams that require a nonlinear model to be solved. First, it is commonly believed that the deep beams are substantially over-reinforced if redistribution of internal forces is not taken into account. Second, many engineers feel that crack width cannot be determined accurately, based on plasticity models.

A nonlinear version of the stringer-panel model is developed in this paper. Besides the panels carry shear at the edges, normal forces are carried by the panels. This more accurate panel appeared necessary for two reasons. First concrete dilates after the onset cracking which influence the distribution of internal forces considerably. To include dilatation, a panel needs degree of freedom perpendicular to the edges. Second, the distributed reinforcement contributes not only to the shear strength but also the normal strength of the material. In the nonlinear model of the panel, the reinforced concrete material can both crack and crush, and the reinforcement can yield and break except the reinforcement of the stringers. Compared to a strut-and-tie model a stringer-panel model is quite simple to make since you do not need to anticipate tension or compression nor to determine the inclination of struts.

### 2.2 Stress-strain relationships

The modified compression field theory (MCFT) was proposed several years ago as a simple analytical model for predicting the load-deformation response of reinforced concrete elements subjected to in-plane shear and normal stresses [6]. The material model considered equilibrium and compatibility conditions within an element in terms of average strains. Local stress conditions at crack locations were also considered. In the MCFT the cracked concrete is treated as a new material with unique stress-strain characteristics. New constitutive relations have been proposed for cracked concrete, based on extensive test data, reflecting significant influences due to compression softening and tension-stiffening mechanisms. The formulations of the MCFT were subsequently incorporated into a nonlinear finite element algorithm [7]. Accordingly, cracked reinforced concrete is treated as an orthotropic material. The solution procedure used is based on a Stiffness formulation, giving good numerical stability and providing much freedom in the definition of material behavior models. The discrete element model consisting of stringers and panels herein is made to reflect the nonlinear behavior of reinforced concrete deep beams by adopting the formulations of the MCFT. Such analyses can be performed by simple modification of existing linear elastic routines and modifications are based on a stiffness of the material.

### 2.3 Material modeling

The stresses in narrow webs of reinforced concrete deep beams can reasonably be considered to be a plane stress. Modified compression field theory is used here. To model nonlinear material response, the constitutive relations contained in the MCFT have been adopted. The concrete compression strength  $f_c$  used in the calculation can be obtained from the measured cylinder compressive strength; the tensile strength of concrete is low compared with the compressive strength and the ductility of concrete under tension is very limited, so the tension strength will be neglected; the reinforcements in both tension and compression are assumed to be rigid-perfectly plastic with yield stress  $f_y$ . Perfect bond is assumed and dowel action is neglected.  $E_c$  and  $E_s$  are the modulus of elasticity of concrete and reinforcement, respectively, and  $E_c$  is calculated according to ACI [8]

$$E_c = 57,000 \sqrt{f_c} \quad (1)$$

in which  $f_c$  is in psi.

### 2.4 Element tangent stiffness matrix

The numerical formulations developed in this paper relied on work previously formulations. To obtain a more accurate estimate for the load-bearing capacity of the reinforced concrete deep beam, the force-displacement relations of the two types of stringer element and panel element, based on virtual work equation, complementary energy principle and the Hellinger Reissner functional, are derived with shape functions for displacement or stress fields on curvilinear co-ordinates or in curvilinear direction. As Figure 2 shows the panel dimensions are

$$\begin{aligned}
 a &= \frac{x_2 + x_3}{2} - \frac{x_1 + x_4}{2} & c &= \frac{x_3 + x_4}{2} - \frac{x_1 + x_2}{2} \\
 b &= \frac{y_3 + y_4}{2} - \frac{y_1 + y_2}{2} & d &= \frac{y_2 + y_3}{2} - \frac{y_1 + y_4}{2}
 \end{aligned} \tag{2}$$

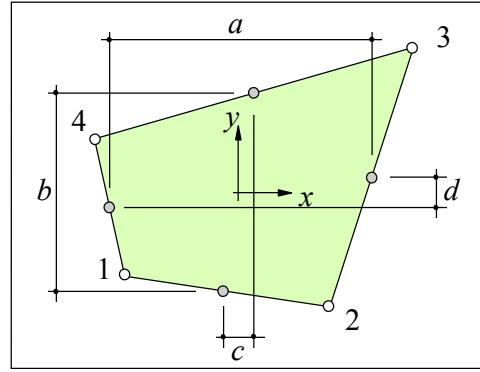


Figure 2. Panel dimensions

For example the variable  $x_i$  is the x co-ordinate of vertex 1. Having determined an appropriate constitutive matrix  $D$ , the stiffness  $k_p$  for a panel element can be evaluated using a direct method [9]. The computations involved can be summarized as

$$k_p = Q P D H A \tag{3}$$

The  $Q$  together with  $P$  embody the equilibrium of the panel, for a rectangular panel

$$\mathbf{Q} = \frac{1}{2t_4} \begin{bmatrix} a^2 & bc & bd - t_4 & -ab & -a^2 & -bc & -bd - t_4 & ab \\ \cdot & 2t_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 2t_4 & \cdot & \cdot & \cdot & \cdot & \cdot \\ -ab & ac - t_4 & ad & b^2 & ab & -ac - t_4 & -ad & -b^2 \\ -a^2 & -bc & -bd - t_4 & ab & a^2 & bc & bd - t_4 & -ab \\ \cdot & \cdot & \cdot & \cdot & \cdot & 2t_4 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 2t_4 & \cdot \\ ab & -ac - t_4 & -ad & -b^2 & -ab & ac - t_4 & ad & b^2 \end{bmatrix} \tag{4}$$

Where  $t_4 = a_2 + b_2$ . For clarity the zero elements of the matrix are represented with dots. The matrices  $A$  and  $H$  embody the kinematic relations of the panel, and they are

$$\mathbf{A} = \begin{bmatrix} \frac{d}{t_1} & \cdot & \frac{b}{t_1} & \cdot & -\frac{d}{t_1} & \cdot & -\frac{b}{t_1} & \cdot \\ \cdot & -\frac{a}{t_1} & \cdot & -\frac{c}{t_1} & \cdot & \frac{a}{t_1} & \cdot & \frac{c}{t_1} \\ -\frac{a}{2t_1} & \frac{d}{2t_1} & -\frac{c}{2t_1} & \frac{b}{2t_1} & \frac{a}{2t_1} & -\frac{d}{2t_1} & \frac{c}{2t_1} & -\frac{b}{2t_1} \\ -\frac{a}{t_2} & \cdot & \frac{a}{t_2} & \cdot & -\frac{a}{t_2} & \cdot & \frac{a}{t_2} & \cdot \\ \cdot & \frac{b}{t_3} & \cdot & -\frac{b}{t_3} & \cdot & \frac{b}{t_3} & \cdot & -\frac{b}{t_3} \end{bmatrix} \quad (5)$$

$$\mathbf{H} = \begin{bmatrix} 1 & \cdot & \cdot & -\frac{c}{a} & \cdot \\ \cdot & 1 & \cdot & \cdot & -1 \\ \cdot & \cdot & 2 & \cdot & \cdot \\ 1 & \cdot & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & \cdot & \frac{d}{b} \\ \cdot & \cdot & 2 & \cdot & \cdot \\ 1 & \cdot & \cdot & \frac{c}{a} & \cdot \\ \cdot & 1 & \cdot & \cdot & 1 \\ \cdot & \cdot & 2 & \cdot & \cdot \\ 1 & \cdot & \cdot & -1 & \cdot \\ \cdot & 1 & \cdot & \cdot & -\frac{d}{b} \\ \cdot & \cdot & 2 & \cdot & \cdot \end{bmatrix} \quad (6)$$

in which

$$t_1 = ab - cd \quad t_2 = \frac{1}{2}(a^2 - c^2) + b^2 - d^2 \quad t_3 = \frac{1}{2}(b^2 - d^2) + a^2 - c^2; \quad (7)$$

In general, the tangent stiffness matrix,  $k_p$  of the panel is not symmetric for two reasons: First, the matrix  $D$  is not symmetric due to the nonlinear material behavior. Second, the equilibrium matrix  $QP$  does not equal the transpose of the kinematic matrix  $HA$ . In this paper, the tangent matrix of the panel,  $k_p$  is not derived analytically but computed by the program. This is necessary because the constitute model contains iterations.

In the stringer-panel model, the concentrated reinforcement and the compressed concrete in the stringers behave linearly, while changing stiffness due to cracking of the tensioned concrete is included. The stiffness  $k_s$  for a stringer element can be evaluated using complementary potential method [9]. The computations involved can be summarized as

$$k_s = T T^T B T F - I B T \quad (8)$$

Due to the sparse matrices involved it is computationally efficient to derive the stiffness matrix analytically instead of computing the matrix multiplications, in which defining matrix  $B$  is

$$B = \begin{bmatrix} -1 & 1 & \cdot \\ \cdot & -1 & 1 \end{bmatrix} \quad (9)$$

$F$  is the flexibility matrix, and it can be observed easily that the matrix is symmetric regardless of the constitutive behavior. In case of a nonlinear model the rotation matrix  $T$  is

$$T = \begin{bmatrix} \cos \alpha & \cos \beta & \cos \gamma & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cos \alpha & \cos \beta & \cos \gamma & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cos \alpha & \cos \beta & \cos \gamma \end{bmatrix} \quad (10)$$

The  $\alpha$  is the angle of the  $x$  axis of the global reference system and the stringer axis. The  $\beta$  and  $\gamma$  are between the stringer axis and the  $y$  and the  $z$  axis, respectively.

### 3. NUMERICAL IMPLEMENTATION

The purpose of the following section is therefore to outline the main features of this analytical method. Further details may be found in Hoogenboom [9]. The numerical implementation of the static approach first requires the discretization of the structure into a few stringer elements and a few panel elements.

#### 3.1 The Number and positions of the newton Cotes integration

During the simulation process of a stringer-panel model, the panel forces are computed for every panel, in every load step and every equilibrium iteration. For computing the stresses in an integration point, a few comments have to be made on the number and positions of the integration. Because four integration points can be positioned such that any numbering results in integration points at the same position, the panel is designed with four integration points, which contain all together twelve strain components of panel. The integration points could be located somewhere in the interior of the panels as is common in the discrete element method. However, since the strain field of the panel is linear in  $x$  and  $y$  directions, strains and stresses at the edges of the panel become larger than at the integration points. As a consequence, at the edge the stresses can become larger than the ultimate material stress,

which is an unsafe approximation with respect to the lower bound theorem of plasticity theory. To be safe the integration points should be chosen at the panel vertices since there the largest strain occurs. However, the strain field in the neighbourhood of the vertices is less accurate especially in panels with a highly nonrectangular shape. So, as a compromise, the integration points are unconventionally positioned at the midst of the edges.

### 3.2 Nonlinear algorithm

Suppose there is a difference between the applied quantities (forces and displacements) and the internal quantities. The difference between an internal nodal displacement and an applied nodal displacement is called a gap. The difference between an internal force and an applied nodal force is called an unbalance. Clearly, the gaps and unbalances should be zero.

If the model is loaded, the gaps and unbalances are present. These differences can be reduced with a computational algorithm. Newton Raphson method is implemented in the stringer-panel model. For this we consider the differences as small extra loads and extra displacements onto the structure. When the structure is assumed to behave linearly the extra internal displacements can be computed with the well known displacement method of linear-elastic analysis: Just assemble the global stiffness matrix, process supports and solve the unknown displacements with the load. The resulting small displacements are added to the internal displacements we already had. Subsequently, the new unbalances will be smaller than before. The gaps are not closed completely due to the computational inaccuracies and the unbalances are not completely eliminated because the structure does not really behave linearly. The remaining gaps and unbalances can be reduced in a next iteration. This is repeated until they are sufficiently small. To compute the complete structural behavior the load is increased in subsequent steps. In every step the above algorithm will adjust the internal nodal displacements in a number of iterations in order to keep up with the load as good as it can. The numerical techniques are shown in Figure 3 for the solution procedure with the stringer-panel model.

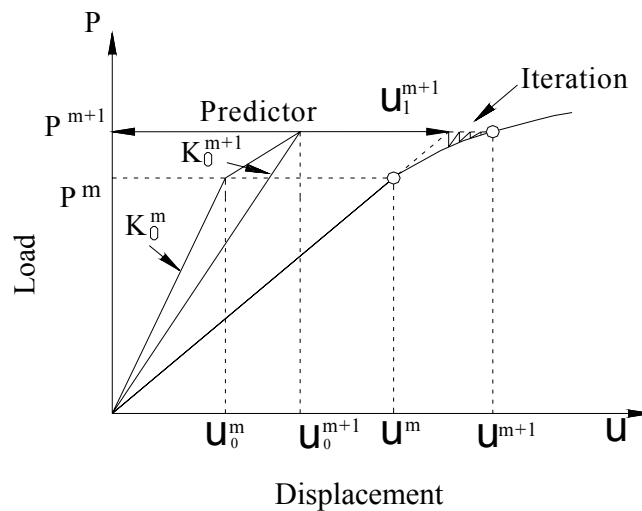


Figure 3. Numerical scheme for incremental analysis



here,  $u^{m+1}$  is the initial displacement for the  $(m+1)th$  minimum load  $P^{m+1}$ ;  $u^m$  is the converged solution at the  $mth$  minimum load  $P^m$ ;  $u^{0m+1}$  is the reference positions at the respective load levels, and  $K^{0m}$  and  $K^{0m+1}$  are all the initial stiffness. Therefore, during the whole process of iterative computation the analysis is carried out incrementally. Apparently, this numerical technique for the incremental analysis is effective because in the iterative solution of a numerical problem, the stability of the numerical solution can eventually be achieved through iterations as long as a predictor is reasonable.

### 3.3 Load control

It is impossible to identify the collapse mode of a given panel with a conventional Newton Raphson iterative analysis under load control since failure of the numerical procedure to converge would be mistakenly interpreted as material failure of the panel. A Newton Raphson iterative analysis under displacement control is not possible yet, because of the inability to control the displacements of several panel points while maintaining a proportional uniform shear and normal biaxial stress [10]. If a conventional load-controlled analysis technique is adopted, it is very difficult to clearly define the ultimate load too. This difficulty can be overcome by adopting a form of arc-length control, whereby, the load factor is allowed to vary during the iterations, and a constraint is applied to limit the incremental displacement [11, 12]. This is sufficient for most situations and it is implemented for the nonlinear analysis of the stringer-panel model.

### 3.4 Convergence criterion

A criterion is necessary to end the equilibrium iterations. Two criteria are implemented in the stringer-panel model which both have to be fulfilled:

The largest unbalance force must be 30 times smaller than the largest of the stringer and panel nodal forces.

The latest displacement gap of the applied displacements must be 30 times smaller than the largest internal nodal displacement.

### 3.5 Smooth relations

The stiffness coefficients initially undergo remarked variation as the behavior changes from uncracked isotropic to cracked orthotropic response or when material starts yielding. When a tangent stiffness matrix is used in the computation of the model behavior, this will often lead to divergence of the iterative process. In this paper, instead of iterating with the local tangent stiffness matrix the initial linear-elastic tangent stiffness matrix is used which can be obtained when the model is not loaded. With this method the computation is very robust, but, the convergence goes slowly and many iterations are required to arrive at an equilibrium state.

## 4. EXPERIMENTAL RESULTS

The stringer-panel model is validated by comparison with test results from eight reinforced

concrete continuous deep beams tested by [13] at Cambridge University in the United Kingdom as shown in Figure 4. These deep beams were to be an extensive research program with the objective to modify the current codes of practice for shear in reinforced concrete continuous deep beams because these codes [8, 14] were based on tests of simply supported deep beams only. The deep beams in the following correlation studies are referred to CDB1, CDB2, CDB3, CDB4, CDB5, CDB6, CDB7 and CDB8 in the original test series. After a shortly description of the observed deep beam behavior, the numerical results with the proposed model are compared with the measured results by Ashour [13].

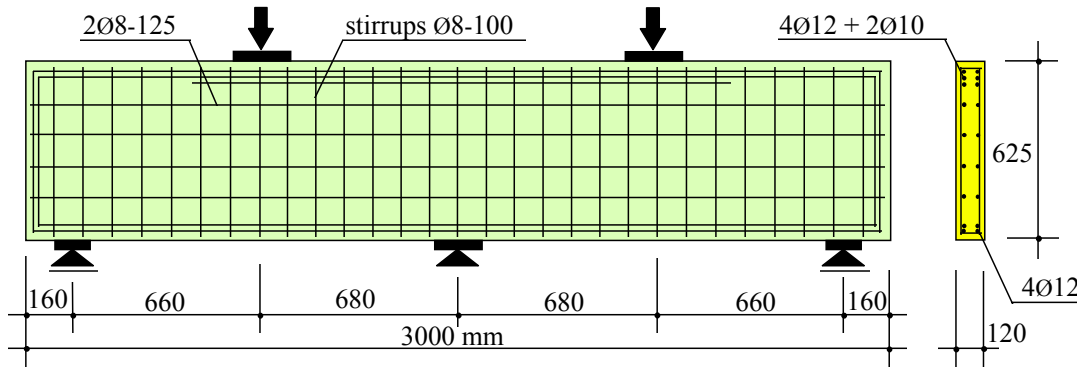


Figure 4. Two-span reinforced concrete deep beam

The overall dimensions of each deep beam are shown in Figure 4. All tested beams have the same length and width: the length is 3,000mm and the width is 120mm. The location of center lines of loads and supports are the same for all test beams. Only the beam depth is varied to obtain two different shear-span-to-depth ratios: for Series I, the depth is 625mm, and for Series II the depth is 425mm (Figure 5). The details of reinforcement for each deep beam are shown in Table 1. The amount of vertical web reinforcement included three levels: none, a low amount, and a large amount. The amount of horizontal web reinforcement studied is none, a low amount, and a large amount. The vertical web reinforcement is closed stirrups and the horizontal web reinforcement is longitudinal bars in both sides of the beam. The main longitudinal top and bottom reinforcements are kept constant for each series except for the last beam (CDB5 and CDB8) in each series, where the amount of top and bottom reinforcements are reduced. All longitudinal bottom reinforcements extended the full length of the beam and through the depth to provide sufficient anchorage.

The material properties of concrete and reinforcement in the deep beam are given in Table 2. All longitudinal top and bottom reinforcements are high-yield ribbed steel bars and the web reinforcement is normal mild steel. The deep beam and the load are symmetrical, so only half the beam is modeled with a stringer-panel model as shown in Figure 5. The stringer-panel model of half the continuous deep beam consists of 7 stringer elements and 2 panel elements.

Table 1: Details of specimen reinforcement

Beam No.	Longitudinal Reinforcement		Web reinforcement	
	Bottom	Top	Horizontal	Vertical
CDB1	4Φ12mm	4 Φ 12+2 Φ 10mm	8 Φ 8mm	29Φ8mm
CDB2	4 Φ 12mm	4 Φ 12+2 Φ 10mm	4 Φ 8mm	15 Φ 8mm
CDB3	4 Φ 12mm	4 Φ 12+2 Φ 10mm	4 Φ 8mm	---
CDB4	4 Φ 12mm	4 Φ 12+2 Φ10mm	---	15 Φ 8mm
CDB5	2 Φ 12mm	2 Φ 12mm	4 Φ 8mm	15 Φ 8mm
CDB6	2 Φ 12+2 Φ 10mm	2 Φ 12+2 Φ 10mm	4 Φ 6mm	29 Φ 6mm
CDB7	2 Φ12+2 Φ 10mm	2 Φ 12+2 Φ 10mm	2 Φ 6mm	15 Φ 6mm
CDB8	2 Φ 12mm	2 Φ 12mm	2 Φ 6mm	15 Φ 6mm

Table 2: Properties of concrete and reinforcement for numerical tests (Mpa)

No.	fc	ft	Ec	fsy	fwy	Es	Ew
CDB1	30.00	4.24	25921.	500	365	2.05E05	2.13E05
CDB2	33.1	4.80	27227.	500	365	2.05E05	2.13E05
CDB3	22.0	3.99	22198.	500	365	2.05E05	2.13E05
CDB4	28.0	4.32	25042.	500	365	2.05E05	2.13E05
CDB5	28.7	4.20	25353.	500	365	2.05E05	2.13E05
CDB6	22.5	4.19	22448.	500	330	2.05E05	2.04E05
CDB7	26.7	4.85	24454.	500	330	2.05E05	2.04E05
CDB8	23.6	4.11	22991.	500	330	2.05E05	2.04E05

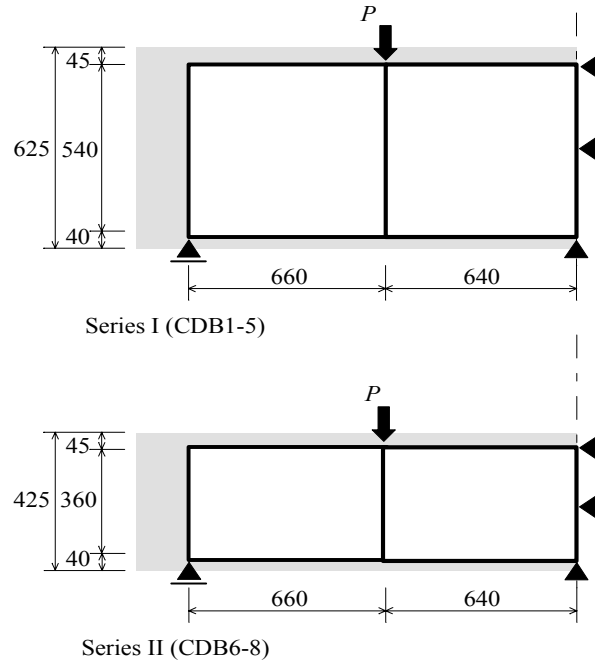


Figure 5. Stringer-panel models of the deep beams

Nonlinear analyses of reinforced concrete deep beams can be achieved by incorporating the element formulations described above into an iterative linear-elastic analysis procedure. Through each iteration, the material stiffness and element stiffness matrices are progressively refined until numerical stability. By adopting this stringer-panel model, the ultimate loads,  $P_c$ , obtained from the numerical analyses are compared with the experimental values,  $P_t$ , [13] in Table 3, in which the abrupt loss of strength after ultimate load attainment indicates concrete crushing. Consequently, the final collapse load is well defined.

Table 3: Calculated failure loads and comparison with experimental results (kN)

No.	$P_c$	$P_t$	$P_c / P_t$
CDB1	530.0	539.6	0.98
CDB2	407.0	466.0	0.87
CDB3	228.0	279.6	0.82
CDB4	352.0	434.1	0.81
CDB5	291.0	402.2	0.72
CDB6	253.0	242.8	1.04
CDB7	218.0	218.3	1.00
CDB8	160.0	188.8	0.85

It can be seen that the numerical results should be accepted from the comparison. Nevertheless, these comparisons of the stringer-panel model with experiments are not enough to establish its validity in all situations. Much more test data is available which should be used to validate or falsify the method and obtain more information on its accuracy.

Also the nonlinear analysis also provides useful information regarding the stringer forces.

## 5. ANALYSES OF NUMERICAL RESULTS

The capabilities and limitations of the procedure presented are reflected in the sample analyses discussed below.

### 5.1 CDB1

The best agreement with the experimental result is obtained with the stringer-panel model. The beam fails at 530.0kN which is very close to the experimental result, 539.6 kN [13].

### 5.2 CDB2

The model collapses at 87% of the experimental ultimate load. CDB2 and CDB1 have equal amounts of longitudinal reinforcement but the amount of web reinforcement is smaller than the one of CDB1. Therefore, CDB2 gets a smaller ultimate load than CDB1.

### 5.3 CDB3 and CDB4

Compared with CDB1 and CDB2, CDB3 has light horizontal web reinforcement and no vertical web reinforcement, CDB4 is just opposite. The web horizontal reinforcement in CDB3 yields at failure [13]. Because of the large different in the reinforcements in the horizontal and vertical directions of the panel, contrasted with the experimental results, CDB 3 and 4 both significant underestimated the collapse that is about 18% and 19% too low, respectively.

### 5.4 CDB5 and CDB8

Except the clear span-to-depth ratio is different for CDB5 and CDB8, the amount of main longitudinal top and bottom reinforcements are reduced, compared with other test deep beams. This involves the yielding of the reinforcement after shear/compression failure in the concrete. Ashour's experiment [13] shows that the top and bottom longitudinal reinforcements yield besides most of the web reinforcement yields. But, indeed, in the stringer-panel model, the concentrated reinforcement and the compressed concrete in the stringers behave linearly, while changing stiffness due to cracking of the tensioned concrete is included. Such a significant swings occur. The numerical failure loads of CDB5 and CDB8 is 72% and 85% of that obtained by the experiment, respectively.

### 5.5 CDB6 and CDB7

The computed results of CDB6 and CDB7 involved only change of web reinforcement, and just before failure major redistribution of the vertical stirrup strain takes place and this

stirrup yields, consequently, there are little different between the analytical and the experimental results. As well, at ultimate loads, associated with CDB8, the Series II of deep beam demonstrated a more ductile response, whereas the Series I of deep beam reflects a more brittle crushing failure because the latter has a smaller clear shear span-to-depth ratio from the load-displacement curves.

To date, numerical models of reinforced concrete are still being improved but they can predict behavior of reinforced concrete reasonably well. A misprediction of about 10% of the ultimate is commonly accepted since the scatter in material properties is of the same order. Numerical difficulties are, however, encountered when the crack distribution in the structural element does not satisfy the assumptions of the crack model. So, a misprediction of under 20% of the ultimate should be accepted. Of all the deep beams tested, the CDB1, 2, 6, and 7 consistently gave almost the same ultimate loads as those observed in the experiments. CDB3 and CDB4 underestimate the ultimate loads by 18% and 19%, respectively. Only CDB5 is equally bad in these instances and additionally gives underestimate of 28%. As mentioned above, the stringer-panel model follows the experiments well with a conservative prediction of the ultimate load. The ultimate loads of the stringer-panel model appear conservative in nature. This can be understood if we consider that the model is in essence an equilibrium system. The reason for the conservative ultimate load can be that both dowel action (bottom stringers) and the contribution of the compression zone (top stringers) to the shear strength, are not included in the model. According to plasticity theory this results in an underestimate of the ultimate load.

The correction studies as mentioned before lead to the following assumptions with regard to the stringer-panel model considered, it is worth to discuss that in this nonlinear computation the reinforcement of the stringers does not yield. Instead, it continues to behave linearly beyond its yield strength. And, in the panels, both the concrete and the distributed reinforcement behave nonlinearly only. Another reason, maybe, is that the panel for simulations has to be able to dilate independently and carry normal stresses as well as shear stresses, therefore, the constitutive model is extended to accommodate modeling of the Poisson's effect.

## 6. CONCLUSIONS

A stringer-panel model is developed for nonlinear analysis of reinforced concrete deep beam. In this model a deep beam is subdivided in panel elements that contain distributed reinforcement and stringer elements that contain concentrated reinforcement. The procedure is based on a stiffness formulation, incorporating constitutive relations for concrete as derived from the modified compression field theory and utilizing only a few discrete elements. The procedure, used in conjunction with a stiffness – based nonlinear analysis algorithm, is found to be numerical stable. Conclusions derived from the work included the following:

In the examination of deep beams previously tested, the nonlinear analyses show that a stringer-panel model with a few elements results in an acceptable accuracy.

According to the stringer-panel model the numerical results are in an underestimation of

the ultimate load. The ultimate load of the stringer-panel model appears conservative in nature because a tested structure somehow finds extra ways to carry forces that are not included in the model.

Because the present stringer-panel model involves only a few numbered elements, there has been no need to tackle “mesh dependency” problem. Consequently, many of the difficulties that, in the presence of softening material, are associated with alternative equilibrium paths have been avoided.

Compared to a strut-and-tie model a stringer-panel model is quite simple to be made since one does not need to anticipate tension or compression nor to determine the inclination of struts.

Newton Raphson is selected in the stringer-panel model, When a tangent stiffness matrix is used in the nonlinear computation, this will often leads to divergence of the iterative process. In the stringer-panel model, instead of iterating with the local tangent stiffness matrix the initial linear-elastic tangent stiffness is used for all iterations. This works almost always and is often fast enough.

If the stringer-panel model is to obtain data on the failure load, a technique such as the arc length method must be used so that numerical failure does not precede structural failure. The performance of the arc length method is superior to iterations under load control in the case of reinforced concrete panels. Not only does the arc length method converge faster, but it also yields more accurate estimates of failure load.

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